HUSO Dubai Mathematics Final Round
7-8th Grade Exam

Student ID: _____ Name: _____

DO NOT OPEN THIS TEST BOOKLET UNTIL INSTRUCTED
Please read the following instructions carefully.

- This question booklet contains 25 questions, 5 sides (including the cover). If you discover any missing pages, please raise your hand.
- You have 60 minutes to complete this exam.
- You must bring a #2/HB pencil or a pen. Rulers, compass, and erasers are also allowed.
- Please bubble in your answers clearly and darkly on the provided answer sheet. Answers not bubbled in will NOT be graded.
- If you need to change an answer, please erase as clearly as possible. Unclear bubbles maybe graded incorrectly.
- NO work in this booklet will be graded. You may use this booklet as extra scratch paper. Additional blank scratch paper will available.
- NO CALCULATORS OR PHONES AND SIMILAR ELECTRONIC DEVICES OF ANY KIND ARE ALLOWED.
- Figures are not necessarily drawn to scale.
- This exam will be graded as follows:
  - Correct answer: +1 pts
  - Incorrect answer: -0.25 pts
  - Blank answer: 0 pts

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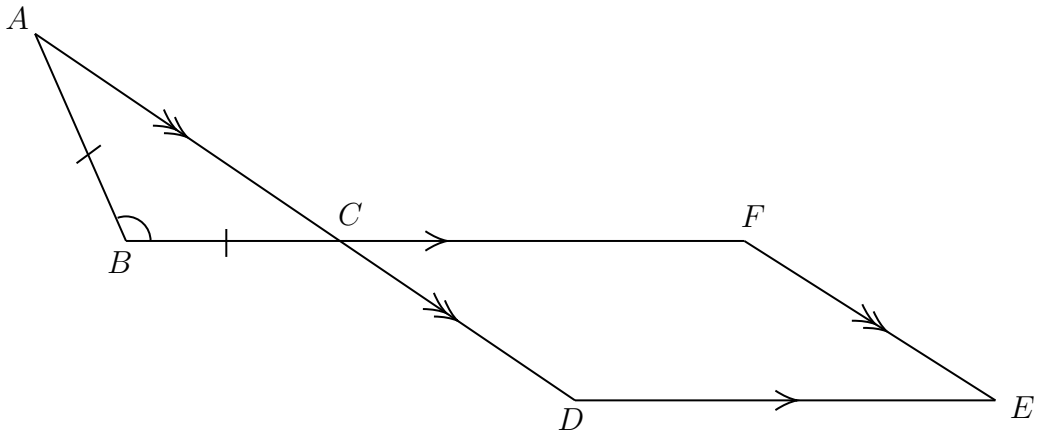
1. An arithmetic sequence starts with $1, 9, 17, \ldots$. What is the 2026th number in the sequence?

   A. 16200 B. 16201 C. 16208 D. 16209 E. 16217

2. In the diagram below, line AD is parallel to line FE, line BF is parallel to line DE, and line segments AB and BC are equal. If $\angle ABC = 110°$, what is $\angle CFE$?



   A. 110° B. 135° C. 145° D. 165° E. 170°

3. A pen contains heffalumps (which stand on 4 legs) and woozles (which stand on 2 legs). Both species only have one head. If you count 26 heads and 86 legs, how many heffalumps are in the pen?

   A. 17 B. 18 C. 21 D. 22 E. 23

4. Let $a = 1011_{10} - 10_{10}$, i.e. the value of $1011 - 10$ in base 10. Let $b = 1011_2 - 10_2$, i.e. the value of $1011 - 10$ computed in base 2, converted to base 10. What is $a - b$?

   A. 992 B. 993 C. 997 D. 998 E. 1000

5. How many obtuse triangles are there with side lengths equal to 1, 2, 3, 4, or 5? A triangle may have sides of equal length, and congruent triangles should only be counted once.

   A. 2 B. 3 C. 4 D. 5 E. 6

6. A fair die is rolled 2 times. What is the probability that you roll 2 numbers in strictly increasing order?

   A. $\frac{1}{3}$ B. $\frac{5}{12}$ C. $\frac{1}{2}$ D. $\frac{7}{12}$ E. $\frac{2}{3}$

7. For how many values of n will an n-sided regular polygon have exterior angles that are integer degrees?

   A. 16 B. 18 C. 20 D. 22 E. 24

8. Farmer John has 84 meters of fence that he can use to surround a rectangular area to make a pen for his cows. What is the area of the largest pen he can make?

   A. 80 B. 152 C. 320 D. 405 E. 441

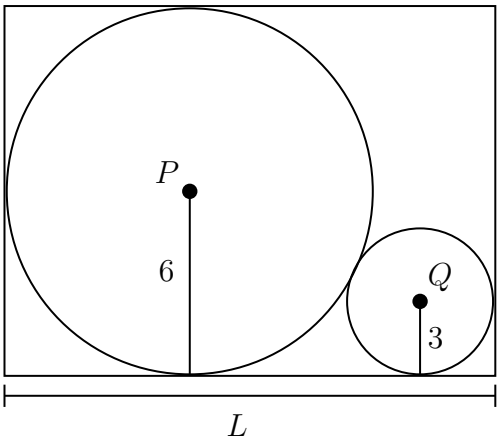
9. Let line A be $y = 3x + 6$. Let line B be parallel to line A and pass through $(2, 2)$, and line C be perpendicular to line A and pass through $(3, 7)$. What is the intersection of line B and line C?

   A. $\left(\frac{18}{5}, \frac{34}{5}\right)$ B. $\left(\frac{15}{4}, \frac{29}{4}\right)$ C. $(4, 7)$ D. $\left(\frac{9}{2}, \frac{19}{2}\right)$ E. $\left(6, \frac{43}{3}\right)$

10. Suppose each ice cream a shop sells consists of one cone, one scoop of ice cream, and one or two toppings. There are 2 cone types, 5 flavors of ice cream, and 10 toppings. An "ice cream combination" consists of a cone, an ice cream flavor, and the topping(s) (order does not matter). How many days does Alice need to visit the shop to guarantee that she has repeated an ice cream combination at least once?

    A. 251 B. 450 C. 451 D. 550 E. 551

11. Two circles, centered at P and Q respectively, are packed into a rectangular box such that the circles are tangent to each other and the sides of the box as depicted below. The radius of the larger circle is 6 and the radius of the smaller circle is 3. What is the length of the box L?



    A. 18 B. $6\sqrt{2} + 9$ C. $3\sqrt{10} + 9$ D. 20 E. $7\sqrt{3} + 9$

12. Two cacti are about to bloom. Each is uniformly likely to bloom any time within the next 15 days, and each will only bloom for 1 day. What is the probability that the two cacti will be blooming at the same time?

    A. $\frac{2}{45}$ B. $\frac{1}{9}$ C. $\frac{29}{225}$ D. $\frac{27}{196}$ E. $\frac{8}{49}$

13. Let $4x^2 + 12xy + 9y^2 = 121$ and $x + y = 4$. If $x, y > 0$, then what is $\frac{y}{x}$?

    A. 1/3 B. 3/5 C. 1 D. 5/3 E. 3

14. Find the largest value a such that 10^a that divides 125!
Note: ! is the factorial operator, so $a! = a \cdot (a-1) \cdot \cdots \cdot (2)(1)$. Ex. $3! = 3 \cdot 2 \cdot 1 = 6$.

A. 25 B. 28 C. 30 D. 31 E. 36

15. Consider the function $f(x) = ax^3 + bx^2 + cx + d$. How many of the following statements are true?

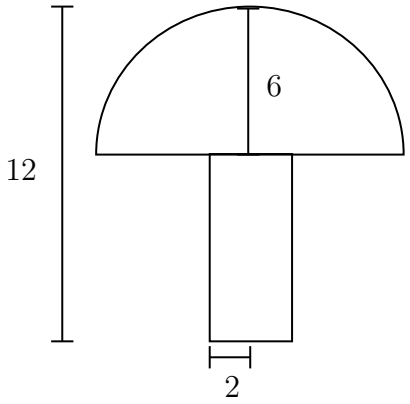
- If a, b, c, d are all non-zero, $f(x)$ is neither odd nor even.
- If $a = 0$ then $f(x)$ must be even.
- If $b = 0, d = 0$ and $a \neq 0, c \neq 0$, then $f(x)$ must be odd.
- There does not exist a, b, c, d such that at most one of a, b, c, d is zero and $f(x)$ is even.

A. 0 B. 1 C. 2 D. 3 E. 4

16. Two rooks are randomly placed on a 8×8 chessboard. What is the probability that the rooks cannot attack each other?
Note: Rooks can attack any square in their row and any square in their column. Two pieces cannot be placed on the same square.

A. $\frac{2}{9}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. $\frac{7}{9}$

17. Fungusville has a giant mushroom sculpture (cross section provided) that consists of a half-sphere balanced on top of a cylinder. The radius of the half-sphere is 6, the total height of the sculpture is 12, and the radius of the bottom of the stem is 2. What is the ratio of the surface area to the volume of the sculpture (including the bottom)?



A. $\frac{43}{78}$ B. $\frac{4}{7}$ C. $\frac{25}{42}$ D. $\frac{17}{26}$ E. $\frac{11}{14}$

18. Let $xy + x + y = 23$. If $|x| < |y|$ and both x, y are integers, then what is the absolute difference between the largest and smallest possible values of x^2y?

A. 45 B. 78 C. 145 D. 189 E. 220

19. Alice spotted 3 birds yesterday. Assume each bird has probability 1/6 of being a blue jay, 1/3 of being a cardinal, and 1/2 of being a duck. Which of the following is the most likely?

    A. Alice saw exactly 1 of each bird B. Alice saw at least 2 ducks C. Alice saw no blue jays D. Alice saw 3 birds of the same type E. Alice saw at most 1 cardinal

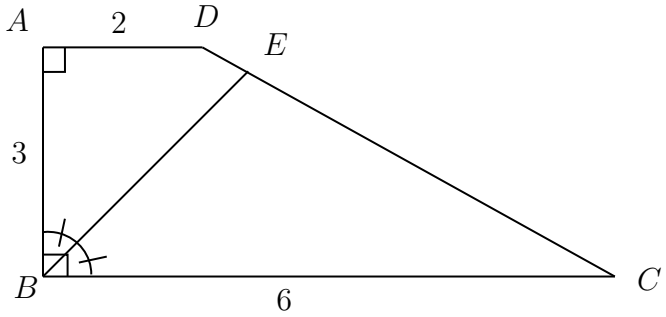
20. What is the coefficient of the x^8 term of $(x+2)^{10}$?

    A. 45 B. 180 C. 248 D. 450 E. 1024

21. Suppose I have a sequence of 7 letters, where each letter can only be C, A, or T. How many ways are there for CAT to appear in the sequence at least once?

    A. 396 B. 399 C. 402 D. 405 E. 408

22. Trapezoid $ABCD$ has side lengths $AD = 2$, $AB = 3$, and $BC = 6$. $\angle DAB = 90^\circ$ and $\angle ABC = 90^\circ$. Line segment BE bisects $\angle ABC$ such that $\angle ABE = \angle EBC$. What is the length of DE?



    A. $\frac{3}{7}$ B. $2\sqrt{2}-\sqrt{5}$ C. $\frac{4\sqrt{13}}{5}-\sqrt{5}$ D. $\frac{5}{7}$ E. $\sqrt{13}-\sqrt{5}$

23. What is the sum of all positive integers $x \leq 100$ such that $3^x - 2^x$ is divisible by 5?

    A. 1300 B. 1859 C. 2250 D. 2499 E. 2550

24. 30 indistinguishable rubber ducks are distributed among 5 people. What is the probability that all of the people get an even number of rubber ducks?

    A. $\frac{51}{682}$ B. $\frac{51}{616}$ C. $\frac{57}{682}$ D. $\frac{57}{616}$ E. $\frac{61}{616}$

25. Let a, b, c, d be the roots of $2x^4-12x^3+25x^2-8x+40$. What is the value of the following expression?

$$\frac{ab+ac+bc}{d}+\frac{bc+bd+cd}{a}+\frac{ac+ad+cd}{b}+\frac{ab+ad+bd}{c}$$

    A. $-\frac{31}{2}$ B. $-\frac{11}{2}$ C. $\frac{11}{2}$ D. $\frac{31}{2}$ E. $\frac{41}{2}$

End of exam.